

Visualizing Fractions: Enhancing Problem-Solving Performance Through Diagrammatic Reasoning in Elementary Mathematics

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Received: April 2025; Revised: July 2025; Published: July 2025

Abstract

This study explores the effectiveness of diagram construction activities, including area proportional and number line representations, in enhancing conceptual understanding and problem-solving performance in fractions among fifth-grade elementary students. Despite the established benefits of visual models, their systematic application in problem-solving contexts remains underexplored. Many elementary students struggle to integrate symbolic and visual representations in fraction problem-solving meaningfully. Using a quasi-experimental design involving 120 students from four public schools in Makassar, the findings reveal that students who participated in diagrammatic reasoning training significantly outperformed those in the control group in both post-test scores and short-term retention. Specifically, the intervention engaged students in constructing area diagrams and number lines to actively visualize fractions. The integration of spatial visualizations through a dual-coding approach proved effective in strengthening symbolic-spatial connections and reducing extraneous cognitive load, with area-proportional strategies emerging as the strongest predictor of accuracy. These results underscore the importance of routinely integrating diagrammatic activities into elementary mathematics curricula and open new avenues for future research using technologies such as eye-tracking and artificial intelligence to support personalized visual learning.

Keywords: Fractions; Diagrammatic Reasoning; Visualization; Problem Solving; Cognitive Load

How to Cite: Sirajuddin, S., Akib, I., & Nasrun, N. (2025). Visualizing Fractions: Enhancing Problem-Solving Performance Through Diagrammatic Reasoning in Elementary Mathematics. *Jurnal Penelitian Dan Pengkajian Ilmu Pendidikan: E-Saintika*, 9(2), 512–524. <https://doi.org/10.36312/e-saintika.v9i2.3139>

 <https://doi.org/10.36312/e-saintika.v9i2.3139>

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INTRODUCTION

Understanding the concept of fractions is a fundamental aspect of elementary mathematics education. Fractions not only serve as a prerequisite for mastering more advanced mathematical operations but are also closely linked to abstract thinking and everyday problem-solving (Charalambous & Pitta-Pantazi, 2007). Unfortunately, numerous studies have shown that elementary students often struggle to grasp both the conceptual and procedural aspects of fraction representation and operations (Siegler et al., 2011; Wijaya et al., 2015). Common misconceptions include interpreting fractions such as "3/4" as simply dividing three by four, or misunderstanding it as three parts of any arbitrary whole. Visual representations such as diagrams and area models have been widely recognized as effective tools to bridge students' conceptual understanding of fractions, especially for those still operating at the concrete cognitive stage (Uttal & Cohen, 2012). In this context, diagrammatic reasoning becomes critical as it involves the use of symbolic and spatial visualization processes to solve

mathematical problems (Stylianou, 2010). However, how fraction visualization through a diagrammatic reasoning approach specifically enhances students' mathematical problem-solving performance remains an area that requires further exploration.

A recurring issue in fraction instruction is students' limited ability to connect symbolic and visual representations of fractions. While many students can memorize the procedures for solving fraction problems, they often lack a true understanding of the underlying concepts (Ng & Lee, 2009). On the other hand, studies such as Fazio & Siegler (2011) have shown that conceptual mastery of fractions is positively correlated with success in advanced mathematics, including algebra. Despite this, the implementation of visual media in instruction often remains sporadic and is rarely integrated into a systematic problem-solving strategy (Hasanah et al., 2021). In contrast, countries like Singapore and Japan have made visualizing mathematical concepts an integral part of their curricula, resulting in significantly improved student performance (Kaur, 2010). These trends highlight an urgent need for systematic, visually-based instructional approaches to strengthen students' understanding and problem-solving skills. Thus, it is crucial to conduct research that integrates diagrammatic reasoning strategies into fraction instruction to address key pedagogical challenges in elementary education.

While previous studies have examined the effectiveness of visual tools or concrete manipulatives in teaching fractions such as fraction blocks, number lines, or pie models (Cramer, Post, & delMas, 2002), few have explicitly investigated the cognitive role of diagrammatic reasoning in enhancing problem-solving performance, particularly in the domain of elementary level fractions. Previous research (see Table 1), for instance, has largely not examined the explicit link between specific types of representations such as area-proportional diagrams or schematic number lines and problem-solving accuracy or long-term retention, especially within the Indonesian educational context. Additionally, most visual interventions are passive, while diagrammatic reasoning actively involves students constructing and manipulating diagrams, which fosters deeper cognitive engagement.

Diagrammatic reasoning is not merely a visual aid; it reflects a way of thinking that is systematic, integrative, and deep (Dörfler, 2003). Moreover, most prior research has focused more on conceptual understanding than on actual performance in solving real-world problems that require higher-order thinking skills. This study seeks to fill that gap by examining how diagrammatic reasoning-based fraction visualization can contribute to improved mathematical problem-solving among elementary students. The focus on diagrammatic reasoning offers added value beyond the use of traditional visual tools by fostering active interactions between symbolic and spatial representations in students' thinking processes.

This study aims to explore the impact of diagrammatic reasoning-based fraction visualization strategies on elementary students' problem-solving performance. The variables examined include: (1) the ability to understand fractions both visually and symbolically; (2) accuracy and efficiency in solving problem-based fraction tasks; and (3) the patterns of diagrammatic representation students employ in their thinking processes. These indicators of accuracy and efficiency are grounded in existing conceptual frameworks on mathematical problem-solving performance (Stylianou, 2010; Mayer, 2014). This study focuses on fifth-grade students in urban public schools

with average to low academic achievement to evaluate the effectiveness of this approach among groups that typically face challenges in mathematics. By emphasizing the development of visual representation strategies and diagrammatic thinking, this research aims to contribute meaningfully to pedagogical practices, curriculum development, and theories of mathematics learning grounded in visualization and multimodal representation.

Table 1. Summarizes the limitations of past approaches and how this study addresses these gaps.

Aspects	Previous Approaches	This study
Types of Visual	Using concrete tools such as fraction blocks, pie charts, or number lines separately	Integrates proportional-area representation and number lines in a single diagrammatic reasoning-based approach
Student Roles	Passively as recipients or users of ready-to-use visual media	Actively creates and interprets diagrams through interactive worksheets
Evaluation Focus	Focus on conceptual understanding	Evaluates conceptual understanding, problem-solving performance, and short-term retention
Research Scope	Generally international studies, rarely in the Indonesian context	Conducted in an Indonesian context with fifth-grade public elementary school students
Teacher Roles	As a procedural presenter or manipulative facilitator	Acts as a facilitator in guiding visual discussions and focusing students' attention on spatial-symbolic relationships

METHOD

Research Design and Type

This study employed a quasi-experimental pretest–posttest nonequivalent group design, commonly used in educational interventions when full randomization is impractical in real classroom settings (Cai et al., 2022). This design allowed the researcher to compare changes in fraction problem-solving performance between a group receiving diagrammatic reasoning-based fraction visualization and a group receiving conventional instruction, while controlling for initial differences using pretest scores. The research framework, as outlined in Table 2, consisted of a pretest phase, eight treatment sessions (2 × 45 minutes per week over four weeks), a posttest, and a retention test four weeks after the intervention. This design was chosen for its effectiveness in evaluating the impact of representational interventions in elementary mathematics learning and is recommended in recent studies on concrete virtual manipulatives (Hussain & Shah, 2024).

Table 2. Nonequivalent group pre-test–post-test quasi-experimental design

Group	Pretest	Intervention	Posttest	Retention (4 Weeks)
Experiment	Fraction Test + Problem Solving Test	Fraction Visualization Based on Diagrammatic Reasoning	Fraction Test + Problem Solving Test	Fraction Test
Control	Fraction Test + Problem Solving Test	Conventional Fraction Teaching	Fraction Test + Problem Solving Test	Fraction Test

Sample and Research Subjects

The target population comprised fifth-grade students from four public elementary schools in Makassar (N = 312). The minimum required sample size was calculated using G*Power for ANCOVA ($\alpha = 0.05$; power = 0.80; $f = 0.25$ medium effect), yielding a requirement of 98 participants. Accounting for a potential 15% attrition rate, 120 students were recruited using a multistage cluster-random sampling technique: (1) selecting two clusters of schools homogeneous in accreditation; (2) designating classrooms as cluster units; and (3) randomly assigning classrooms to either the experimental group (n = 60) or the control group (n = 60). This procedure aligns with stepwise sampling recommendations for quasi-experimental educational research (Winke & Reinders, 2023). The main characteristics of the subjects' age (10–11 years), prior mathematics achievement, and gender distribution were confirmed to be equivalent across groups using χ^2 and independent t-tests before the intervention. Detailed demographic data including participants' mean age, mean prior math achievement scores, and gender distribution are provided in Table 3.

Table 3. Detailed demographic data

Group	Number of Students	Average Age (years)	Prior Math Score (average, out of 100)	Male	Female
Experimental	60	10.4 ± 0.5	71.3 ± 8.2	30	30
Control	60	10.3 ± 0.6	70.8 ± 7.9	31	29
Total	120	10.4 ± 0.5	71.0 ± 8.0	61	59

Instruments and Procedures

Quantitative data were collected using two instruments: 1) The Fraction Understanding Test (10 multiple-choice items) measured conceptual knowledge using area, length, and set representations. It was adapted from the Fraction Diagnostic Test by Cramer et al. (2002) and contextualized to the Indonesian curriculum. Content validity was confirmed by three experts (CVI = 0.92); Rasch analysis indicated infit values between 0.83 and 1.18, with person reliability of 0.87. 2) The researcher developed the Fraction Problem-Solving Test (5 contextual open-ended questions) based on diagrammatic reasoning indicators (Stylianou, 2010). Internal reliability was $\alpha = 0.82$, within the acceptable range recommended by Cronbach's guidelines (Taber, 2018). Sample items from the Fraction Problem-Solving Test and examples of categorized student responses (iconic, area-proportional, schematic):

Alya has 8 coloring pencils. She uses $\frac{5}{8}$ of them to draw. How many pencils does Alya not use?

Additionally, an example of the visual worksheet used by students:

Divide the following area into $\frac{3}{4}$ parts.

The research procedures included: (a) teacher and assistant orientation (including detailed training on diagrammatic reasoning strategies and worksheet implementation); (b) simultaneous pretesting; (c) eight intervention sessions: the experimental group used interactive worksheets involving diagram construction and interpretation tasks, while the control group followed textbook-based explanation and practice; (d) posttesting; and (e) a retention test. All sessions were video-recorded to support qualitative triangulation of students' diagrammatic representation patterns.

The difference in cognitive engagement between the experimental group (interactive activities) and control group (passive textbook tasks) is acknowledged as a potential confounding factor and is discussed further in the discussion section.

Data Analysis

Pretest, posttest, and retention scores were analyzed quantitatively using ANCOVA (covariate = pretest score) to examine between-group performance differences and calculate the partial η^2 effect size. Assumptions of normality and homogeneity were verified using the Shapiro-Wilk and Levene's tests. Intra-group improvement was tested using paired t-tests with Bonferroni correction. Additionally, students' diagrammatic responses were inductively coded into three strategy categories (iconic, area-proportional, and schematic), with intercoder reliability assessed at $\kappa = 0.86$, indicating strong agreement. The relationship between strategy type and accuracy was analyzed using a χ^2 test. This mixed-methods analytic approach aligns with contemporary practices in evaluating educational interventions, which combine inferential statistics and content analysis (Kaya & Canga, 2024), thereby strengthening the interpretation of findings related to improving fraction problem-solving performance through diagrammatic reasoning.

Ethical Statement

This classroom-based study with fifth-grade students in Makassar (ages 10–11) adhered to standard research ethics for work with minors. Prior to any activity, schools and teachers granted permission; parents/guardians provided written consent; and students gave age-appropriate assent. Participation was voluntary; students could withdraw at any time without penalty. No incentives were offered. Data collected (fraction tests, problem-solving tasks, and video recordings of lessons) were used solely for research, stored on password-protected devices with restricted access, and de-identified before analysis; reporting is in aggregate only. Video data were obtained with explicit consent and were coded to study diagrammatic strategies; no images or clips will be shared publicly or used for identification. The control group received business-as-usual instruction, and the intervention posed no more than minimal risk. Procedures complied with relevant institutional and school policies on child data protection and with accepted guidelines for educational research.

RESULTS AND DISCUSSION

Improvement in Conceptual Understanding of Fractions

Quantitative findings presented in Table 4 and Figure 1 shows a substantial increase in the experimental group's average fraction understanding score, rising from 45.2 (SD = 8.1) to 77.8 (SD = 6.4), while the control group improved more moderately from 44.7 (SD = 9.0) to 60.3 (SD = 7.2). ANCOVA results (Table 6) confirmed a significant treatment effect, $F(1, 117) = 56.48$, $p < .001$, partial $\eta^2 = 0.15$, indicating a substantial contribution of diagrammatic reasoning-based visualization to conceptual development in fractions. This pattern suggests that diagrammatic representations strengthen the numeric spatial linkage in students' working memory, enabling more meaningful reconstruction of part-whole relationships and the relative value between numerators and denominators.

conceptual outcomes ($N = 10,562$). Research by Kaminski et al. (2018) demonstrated that area models and number lines improve fraction discrimination and cross-representational transfer. These consistent findings reinforce the idea that integrating spatial elements facilitates faster mental mapping between symbolic representations and quantities. The stronger achievement effects observed here, compared to a Barbosa and Vale (2021) study reporting $g = 0.30$, may be attributed to the intensity of sessions (8×45 minutes) and the explicit focus on diagrammatic strategies rather than passive visual media. A limitation of this study is the absence of cognitive load measurement, which may mediate the visualization effect.

Table 5. The ANCOVA test of posttest score

Source	SS	Df	MS	F	p	Partial η^2
Group (covar=Pre)	3120.4	1	3120.4	56.48	<.001	0.15
Error	6465.8	117	55.3			
Total	9586.2	118				

Problem-Solving Performance

In addition to conceptual understanding, the experimental group showed a 32.6 percentage-point increase in fraction problem-solving accuracy, far surpassing the 15.4-point gain in the control group. A gain-score test indicated a significant difference, $t(118) = 8.12$, $p < .001$. Average solution time decreased by 41% in the experimental group compared to just 19% in the control group. This suggests that effective diagrams not only expand declarative knowledge but also help automate proportional inferences during the solution process. These findings are supported by the onto-semiotic framework of diagrammatic reasoning, which highlights the dominant role of visual inference in structuring mathematical problem-solving steps. Empirically, a study on visual puzzles in Spain found a correlation of $r = 0.52$ between the quality of diagrammatic schemes and answer accuracy (Stylianou, 2010). However, our study yielded a larger effect size (Cohen's $d = 1.07$), possibly due to the structured intervention context and teacher scaffolding, as opposed to naturalistic observational studies. One limitation is the uncontrolled factor of students' intrinsic motivation, which may have moderated performance outcomes.

Four-Week Retention Post-Intervention

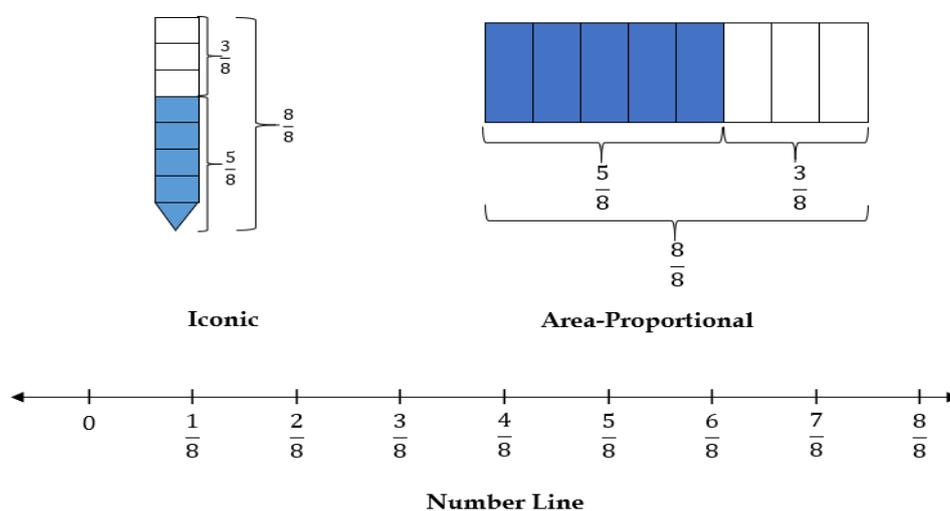
Retention tests showed a slight decline in the experimental group's scores (-3.2 points), yet the scores remained significantly higher than the control group's (-5.2 points). ANCOVA on retention scores remained significant, $F(1, 117) = 32.97$, $p < .001$. This suggests the benefit of sustained memory through dual verbal-visual encoding. The superiority of graphics over text for long-term retention was recently reported by Ciccione et al. (2025), with an effect size of $g = 0.48$. Similar to our findings, they emphasized the role of image-concept bonding in reducing memory decay. The residual score difference between experimental and control groups (see Table 4) in our study (19.5 points) exceeded the typical gap observed in general visual literacy studies (~ 12 points), possibly due to the inherently visual nature of mathematics and repeated diagrammatic practice. One limitation is the relatively short four-week retention interval; longitudinal studies are needed to assess the stability of effects over a semester or longer (as synthesized in Table 6).

Table 6. Analysis of the Types of Fraction Problems Retained

Types of Fraction Problems	Proportion of Experimental Students Who Answered Correctly (%)	Proportion of Control Students Who Answered Correctly (%)	Dominant Retention Status
Conceptual – Area Model	87	52	Maintained
Conceptual – Number Line	81	47	Maintained
Procedural – Simple Fraction Operations	74	61	Tends to be Maintained
Procedural – Mixed Fractions	63	50	Tends to be Forgotten
Contextual Applications	69	41	Maintained
Conversion Between Representations	58	32	Forgotten

Diagrammatic Strategy Patterns and Accuracy

Qualitative analysis identified three strategy types: iconic (18%), area-proportional (52%), and schematic number line (30%). A chi-square test, $\chi^2(2, 58) = 29.41$, $p < .001$, indicated a significant relationship between strategy type and accuracy, with 87% of correct answers originating from area-proportional representations. This pattern is consistent with expert teacher strategy studies, which emphasize area models as the most intuitive numeric-spatial bridge for students. A study in Peru on fraction problem-solving also found a dominance of area models among high-achieving students (Noriega et al., 2024). However, unlike our results, they reported equal contributions from number lines and area models. This discrepancy may stem from curriculum differences number lines are introduced earlier in Peru (from Grade 4), while exposure in Indonesia tends to occur later. A limitation of our analysis is the potential subjectivity in categorizing strategies, despite high intercoder reliability ($\kappa = 0.86$); incorporating eye-tracking could improve the accuracy of identifying students' internal visual processing.

**Figure 2.** Diagrammatic Strategy Type

The substantial improvement in conceptual understanding and problem-solving performance observed in the group trained with diagrammatic reasoning underscores the dual function of visual representation: as cognitive scaffolding and as a “second language” complementing formal mathematical symbols. Theoretically, these findings reinforce both Dual-Coding Theory (Paivio, 1991) and the Cognitive Theory of Multimedia Learning (Mayer, 2014), which position the verbal-spatial channels as parallel conduits for information processing. However, our study contributes domain-specific evidence in the area of fractions, a topic that has received relatively limited attention in this context. Supporting this, Lee and Yeo (2024) demonstrated how the Dynamic Ruler tool allows teachers and students to “negotiate” unit sizes through linear drag-and-stretch actions, rendering the concept of fraction-as-measure visually salient. In younger age groups, Cutting (2021) reported that mental rotation and object visualization abilities significantly predict fraction understanding among six- and seven-year-olds, indicating that spatial foundations are at play even before symbolic forms like “ $\frac{1}{2}$ ” are introduced. The strong correlation we observed between area-proportional strategies and the transfer of “part-whole” understanding suggests that diagrams serve as conceptual bridges, reducing abstraction barriers while enriching the spatial image bank and fostering more durable proportional schemas.

Empirically, the treatment effect size (partial $\eta^2 = 0.15$) in our study exceeds that of Barbosa and Vale’s (2021) visual module on cognitive load ($g \approx 0.30$), which primarily involved static images. This difference may be attributed to the active role of students as diagram creators, aligning with findings by Rellensmann, Mayer, & Mayer (2025), who noted that self-generated, mathematically accurate drawings reduce extraneous cognitive load and enhance real-world problem-solving performance. This constructive approach also resonates with the work of Yuliandari et al. (2024), who emphasized the effectiveness of teachers’ commognitive frameworks in integrating verbal, symbolic, and contextual representations when teaching fractions. By contrast, studies focusing on number line routines published in nationally accredited journals reported moderate gains ($d \approx 0.45$), though more stable across varying ability levels, likely due to the consistent nature of daily practice. This suggests that the intensity and continuity of practice moderate the strength of diagrammatic effects. Thus, combining routine number line exercises (to build procedural fluency) with active diagram projects (to trigger conceptual surges) could provide the most synergistic benefit across diverse learner profiles.

The practical implications point toward classroom designs that engage students in constructing, verifying, and revising their own diagrams while reflecting on the most efficient visual strategies. Yuliandari et al. (2024) assert that variation in representations can reduce math anxiety, particularly for students with low spatial profiles. Our findings are consistent with a study by Wang et al. (2024), which identified a mediating chain involving spatial ability, math anxiety, and fraction performance accounting for approximately 30% of the variance in achievement. To capture the dynamics of visual attention during diagram construction, future research could incorporate eye-tracking and real-time cognitive load metrics, as recommended by Gonnermann-Müller et al. (2024) in their study on AR visual guidance. In addition to extending retention intervals to a full semester, longitudinal research should also examine transfer effects to other rational number domains (decimals, percentages) and explore diagram generators. Such developments would enable diagrammatic

reasoning-based instructional innovations to be adaptive, sustainable, and inclusive across diverse educational contexts.

CONCLUSION

This study demonstrates that utilizing diagrammatic reasoning-based fraction visualization strategies significantly enhances elementary students' conceptual understanding and problem-solving performance, surpassing both conventional approaches and the use of static images. Interventions that position students as diagram constructors rather than passive observers strengthen the connection between symbolic and spatial representations, reduce extraneous cognitive load, and foster proportional schemas that persist for at least four weeks post-instruction. These findings fulfill the research objective of testing the effectiveness of diagrams as both conceptual bridges and cognitive scaffolding mechanisms, and support the hypothesis that constructing diagrams yields greater impact than passive visualization. Practically, the results highlight the importance of designing classroom tasks that require students to create, evaluate, and revise area diagrams or number lines under teacher guidance informed by the multiple representations framework. Routine implementation of this strategy has the potential to reinforce conceptual resilience in fractions, reduce math anxiety, and promote transfer to other rational number topics such as decimals and percentages. For curriculum developers, integrating diagrammatic worksheets and constructive visual tasks should be prioritized in textbooks and digital learning platforms.

Future research is recommended to: (1) extend retention intervals to assess the stability of effects over a semester or longer; (2) implement interventions across diverse contexts rural, multicultural, and inclusive classrooms to evaluate generalizability; (3) combine eye-tracking and real-time cognitive load measurements to map visual attention dynamics during diagram construction; and (4) explore AI-powered technologies that generate adaptive diagram recommendations based on students' spatial profiles. These efforts are expected to enrich visual-based mathematics learning models, underscore the relevance of diagrammatic reasoning in 21st-century STEM literacy, and pave the way for inclusive and sustainable instructional innovations.

RECOMMENDATIONS

To deepen and broaden the application of diagrammatic reasoning in fraction instruction, this study recommends several strategic follow ups. These include conducting a year, long longitudinal study combining number line routines and active diagram construction to assess long-term effects and transfer to other rational number domains, integrating eye-tracking and real-time cognitive load measurements to optimize interventions based on students' visual attention, developing AI-based tools that personalize diagram types according to students' spatial profiles and math anxiety, and replicating interventions in diverse geographic and sociocultural settings to evaluate generalizability and curricular alignment. Anticipated challenges such as limited instructional time, diverse spatial abilities, teacher readiness, and technological infrastructure must also be addressed through solutions like online diagram homework, differentiated instruction, targeted teacher training in visual scaffolding and multimodal assessment, and phased collaboration with ed-tech

partners. Addressing these elements can help future research build robust empirical foundations and promote adaptive, effective, and inclusive fraction learning.

Author Contributions

All authors have read and agreed to the published version of the manuscript.

Funding

This research did not receive any external funding.

Acknowledgement

The authors extend their sincere gratitude to the principals, teachers, and students of SD Inpres Andi Tonro, SD Inpres Minasa Upa, SD Negeri Kompleks IKIP, and SD Inpres Bertingkat Bara-baraya II in Makassar City, South Sulawesi, Indonesia, for their permission, administrative support, and technical assistance during the data collection process. We also deeply appreciate the constructive feedback from our colleagues in the Mathematics Education Department at Universitas Muhammadiyah Makassar for their valuable contributions to the methodological discussions and instrument validation.

Conflict of interests

The authors declare that there is no conflict of interest. No external party had any role in the study design; data collection, analysis, or interpretation; manuscript writing; or the decision to publish the findings. Furthermore, this article has not been previously published, nor is it under consideration, submitted, or in process in any other journal.

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